Robinson Crusoe decided that he will spend exactly 10 hours per day searching for food. He can spend this time looking for coconuts or fishing. He is able to catch 2 fish or find 3 coconuts in 1 hour. Robinson's utility function is $U(F, C) = 3F^{0.6}C^{0.3}$*, where F is his daily consumption of fish and C – of coconuts.*

a) How many fish should Robinson catch and how many coconuts should he find so that his consumption maximizes his utility?

$$\frac{F}{2} + \frac{C}{3} = 10$$

$$\text{or } 3F + 2C = 60$$

Note: We get F divided by 2 and C divided by 3 from the information that Robinson is able to catch 2 fish or 3 coconuts in 1 hour. The value 10 is from the information that he works 10 hours per day.

$$MRS = \frac{3 * 0.6F^{-0.4} * C^{0.3}}{3 * 0.3C^{-0.7} * F^{0.6}} = \frac{2C}{F}$$

We know that :

$$\frac{3}{2} = MRT_{fc} = - MC_f / MC_c = - P_f / P_c = - MU_f / MU_c = MRS_{fc} = \frac{2C}{F}$$

MRT is in relation to producers whereas MRS is in relation to consumers.

$$MU_f / P_f = MU_c / P_c$$

$$\frac{2C}{3} = \frac{F}{2}$$

$$F = \frac{4C}{3} \text{ or } 3F=4C$$

Put this equation into 3F + 2C = 60

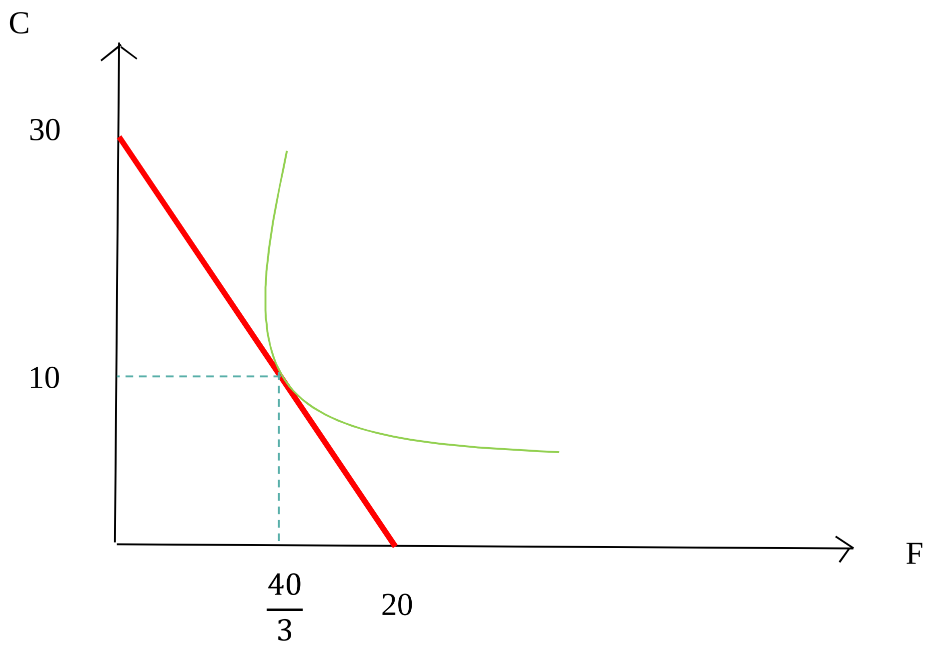
$$\frac{3(4C)}{3} + 2C = 60$$

$$C = 10$$

$$F = \frac{4 * 10}{3} = 13.3$$

From the calculations above we get 40/3 Fish and 10 Coconuts ⇒ $U = 3F^{0.6}C^{0.3} = 28.3$

b) Illustrate the equilibrium with a graph.



One day a native inhabitant of another island arrives on Robinson's island. The visitor offers Robinson trade of 3 fish for 1 coconut. However, trade is not free, it costs 1 fish (that must be paid prior to the exchange).

c) Will Robinson decide to trade? Justify your answer and provide a graph.

Since Robinson has comparative advantage in C, he will specialize in coconuts ⇒ maxC = 10*3 =30

However, if he wish to trade with native inhabitant, he has to pay 1F before the trade ⇒ he cannot spent all his time just on picking coconuts, but he has to catch at least one fish ⇒ he will spend 0.5h on F and 9.5h on C ⇒ maxC = 0.5*1 + 9.5*3 =28.5, where 3 is his productivity in C. but he cannot pick 0.5C ⇒ maxC=28

If Robinson catch only fish, his maxF=10*2-1=19, i.e. 1is the transaction cost and 2 is his productivity in F. However, he may have more F through the trade rather than catch them directly ⇒ If he picks coconuts and exchange them for fish, the maxF=28*3=84, i.e. 3 is the exchange rate

His new PPF is the following:

$$F + 3C = 84$$

where (F+3C) describes the exchange rate with native inhabitant, while 84 is maxF

In other words,

$$\frac{F}{9} + \frac{C}{3} = 9.5$$

here $\frac{C}{3}$ means his productivity 3C per hour since he will specialize in coconuts, $\frac{F}{9}$ means his exchange rate 3C*3 since he will buy F, and 9.5=10-0.5 means the time he may spend on picking coconuts since 0.5h he has to spent on fishing in order to cover transaction cost.

His new MRT=$\frac{1}{3}$ (the previous was MRT=$\frac{3}{2}$)

MRS=MRT

$$\frac{2C}{F} = \frac{1}{3}$$

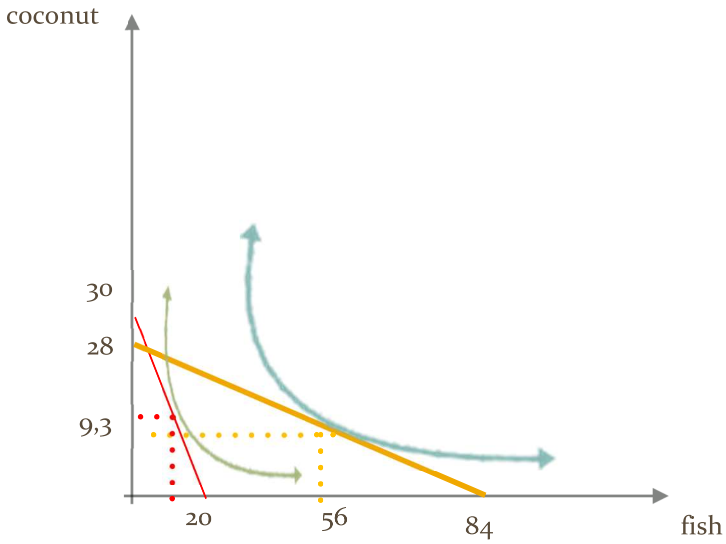
$$\boldsymbol{F = 6C}$$

$$\Downarrow$$

$$6C + 3C = 84$$

C*=84/9=9.3

F*=56



New utility U = $3F^{0.6}C^{0.3}$ = 65.6 is higher than the old one ⇒ Robinson will choose to trade.

d) What will Robinson produce?

Due to comparative advantage he will produce coconuts.

e) What will Robinson consume?

His preferences (Cobb-Douglas) enforce him to consume both goods. As in graph and calculations above he will consume 56 units of fish and 9.3 coconuts